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Rough Anti-Ideals in Semirings

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Abstract

In this paper, the notion of rough anti-ideals and proposal the of an extension of the conventional theory in semirings are studied. We provide illustrative examples to clarify our definitions and explore structural properties related to rough anti-ideals. We examine the concept of rough anti-ideals in semirings in detail and contribute to the theoretical development of rough set theory as applied to the algebraic structure of semirings. We use the congruence relations and thoroughly analyze them to understand the properties of left and right anti-ideals and the associated algebraic structures. We try to find ways to advance theoretical and applied research by clarifying the properties and relationships between rough anti-ideals and semirings. Despite these advancements, there remain numerous properties and directions yet to be explored. Investigating the behavior and applications of rough anti-ideals in diverse mathematical and real-world contexts offers significant potential for further study. Additionally, examining the connections between rough anti-ideals and other algebraic structures could lead to new discoveries and foster cross-disciplinary collaboration.By utilizing congruence relations and thoroughly analyzing the properties of left Furthermore, we examine the relationships between algebraic structures involving rough anti-ideals and semirings, focusing on their theoretical foundations. We aim to clarify these concepts and investigate their potential applications in theoretical and practical contexts.

Key words: Semirings, Anti-Ideals, Lower Approximation, Upper Approximation, Rough Set.

المثالي المضاد الخشن في شبة الحلقة

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الملخص

في هذه الورقة البحثية، ندرس مفهوم المثاليات المضادة الخشنة، ونقترح توسيعًا للنظرية التقليدية في شبه الحلقات. نقدم أمثلة توضيحية لتوضيح تعريفاتنا واستكشاف الخصائص الهيكلية المتعلقة بالمثاليات المضادة الخشنة. ندرس مفهوم المثاليات المضادة الخشنة في أشباه الحلقات بالتفصيل، ونساهم في التطوير النظري لنظرية المجموعات الخشنة كما تُطبق على البنية الجبرية لأشباه الحلقات. نستخدم علاقات التطابق ونحللها بدقة لفهم خصائص المثاليات المضادة اليسرى واليمنى، والهياكل الجبرية المرتبطة بها. نسعى لإيجاد سبل لتطوير البحث النظري والتطبيقي من خلال توضيح الخصائص والعلاقات بين المثاليات المضادة الخشنة وأشباه الحلقات. على الرغم من هذه التطورات، لا تزال هناك خصائص واتجاهات عديدة لم تُستكشف بعد. يوفر البحث في سلوك وتطبيقات المثاليات المضادة الخشنة في سياقات رياضية وواقعية متنوعة وإمكانات كبيرة لمزيد من الدراسة. بالإضافة إلى ذلك، فإن دراسة الروابط بين المُثُل العليا المُضادة الخشنة والهياكل الجبرية الأخرى قد تُؤدي إلى اكتشافات جديدة وتُعزز التعاون بين التخصصات. من خلال استخدام علاقات التطابق والتحليل المُتعمق لخصائص اليسار، ندرس العلاقات بين الهياكل الجبرية التي تتضمن المُثُل العليا المُضادة الخشنة وأشباه الحلقات، مع التركيز على أسسها النظرية. نهدف إلى توضيح هذه المفاهيم والبحث في تطبيقاتها المُحتملة في السياقات النظرية والعملية.

الكلمات المفتاحية: شبه الحلقة، المثالي المضاد، التقريبات من اعلي، التقريبات من اسقل، المجموعة الخشنة.

Introduction

Pawlak [1] introduced rough set theory in 1982 as a new mathematical tool for dealing with uncertainty. This

theory is based on two key concepts: the upper approximation and the lowers approximation, which are the union of all equivalence classes that are subsets of a given set, and the union of all equivalence classes that intersect the set non-trivially respectively. Over time, rough set theory has captivated researchers working on real-life applications and theoretical developments. It has been expanded in various fields and applied it in many areas. Building on Pawlak's work, Biswas and Nanda [2] given the notation of rough subgroups as a new concept. B. Davvaz [3] proposed the concept of rough subrings concerning ideals, while Kuroki [4] developed the idea of rough ideals in semigroups. Yao [5] explored lattice structures through rough set theory. B. Davvaz [6] further extended these ideas by using the concept of (rough prime, rough primary) ideals in commutative rings. Through previous research's, we tried to apply and extend the concept of the rough set-in rings such as in [7] studied connections between rough set theory and ring theory, and, along with Shlitet [8], investigated rough ideals in local rings. Later, Abdunabi and collaborators [9] introduced the concept of rough BO/BH/Z algebras, and in another study [10], Abdunabi explored further links between rough sets and ring theory. Vandiver [11] in 1934 introduced the concept of semirings. An algebraic structure with two associative binary operations, where one operation distributes over the other called a semiring. For example, the unit interval on the real line (I, max, min) forms a semiring, with (0,1) as the additive and multiplicative identities, respectively. Although semirings generalize rings, the ideals of semirings do not necessarily align with those of rings. To address this, Henriksen [12] introduce the k-ideals in semirings to derive peers of ring results. More recently, Abdunabi and Shlitite [13] studied rough pseudo-anti-ideals in anti-rings, presenting properties of their upper and lower approximations.
Our work to introducer the concept of rough anti-ideals in semirings, beginning with the definitions of left and right anti-ideals. Moreover, to extend our notion to defined the rough anti-ideals in semigroups as a new notation. Several nontrivial examples are provided to illustrate the concept and offer deeper insights into its structural properties. Finally, we demonstrate the connections between semiring algebraic structures and rough anti-ideals. We hope this work will enhance the understanding of the theoretical implications of these concepts and inspire future applications.

Preliminaries

This section presents key definitions and foundational results previously submit by majors in this field, which are essential for deriving the new results introduced in this paper.

Suppose that $U\neq\emptyset$ and ~ is an equivalence relation on $U$and $U/\sim$ is the family of all equivalent classes of ~ and $[x]_\sim$ is an equivalence class in ~ containing an element $x\in U$. The Xc define as the complementation of X in U for any X $\subseteq$ U.

Definition 2.1[15]: A semiring $(R,+,\bullet)$ is a nonempty set R with two defined operations (+,*) satisfying the following conditions:

1- (R, +)is a commutative monoid with identity element 0;
2- (R, •)is a monoid with identity element $1 \neq 0$;
3- $r * (s + m) = r * s + r * m$ and $(r + s) * m = r * m + s * m$ $\forall r,s,m\in R$;
4- $0 * r = 0 = r * 0$ $\forall r\in R$.

Remark 2.1. A semiring may have an identity 1 defined by $(1 * r)=x=(r * 1)$ and a zero 0, defined by $(1+r)=r=(r+1)$ $\forall\, r \in R$.

From now on, we write for semirings.

Definition 2.2: let R be a semiring. A subset $I\neq\emptyset$ of R is called a (left, right) ideal respectively if $\forall r,s\in I$ and $r\in R\Rightarrow r+s\in I$ and $r * s\in I$ and (resp $r * s\in I$.) and I is called a two-sided ideal or simply ideal of R, if I is both left and right ideal of R.

Definition 2.3: Let $\sim$ be an equivalence relation on a universal set U. An upper approximation and lower approximation of a set S concerning ~ are $\overline{\sim S}=\{a\in U:[a]_\sim\cap S\neq\emptyset\}$ and $\underline{\sim S} = \{a \in U: [a]_\sim \subseteq S\}$ respectively. If $BS_\sim = \overline{\sim S} - \underline{\sim S} = \emptyset$, then S is an exact (crisp) set and if $BS_\sim \neq \emptyset$, S is called rough.

Definition 2.4: Let I be an Ideal of a ring R. Call a is congruent of b mod I and write $a \equiv b(mod\ I)if\ a - b \in I(1)$.

Remark 2-2: The relation (1) is an equivalence relation.

Definition 2.5: Let the universal set U be equal the ring R, Defined the upper and lower approximation of S with respect of I as: $\overline{I(S)}=\cup\{ r\in R :(r+I)\cap S\neq\emptyset\}$, $\underline{I(S)}=\cup\{ r\in R :r+I \subseteq S\}$, respectively. And, $BS =\overline{I(S)}-\underline{I(S)}$ the boundary of S $with\ respect\ of\ I$.

Definition 2.6. A pair$(U,\sim)$ with$U \neq \emptyset$ and ~is an equivalence relation on U is called an approximation space

Definition2.7: The ideal M in a ring R is a maximal if $M \neq R$ and it is just an ideal strictly containing $M\ is\ R$.

Remark 2-3: Note that, every maximal ideal of R is prime if R is a commutative ring with identity.

Definition 2.8. Let $(U, \sim)$ be approximation space. A pair $(A, B) \in P(U) \times P(U)$ is a rough set in if and only if $(A, B) = (\overline{\sim S}, \underline{\sim S})$ for some $\subseteq U$. If A and B are any two subsets of R, then $AB = \{ab : a \in A, b \in B\}$.

Antiideals Of A Semiring

This section considers the concepts of anti-ideals in semirings. Some properties and significance are studied.

Definition 3.1. Let $(R, +, *)$ be a semiring and $\emptyset \neq I \subseteq R$. Then I is:

1- a left anti-ideal of R if $RI \cap I = \emptyset$.
2- a right anti-ideal of R if $IR \cap I = \emptyset$.
3- an antiideal of R if it is a left and right ideal of R

Example 3.1. Let $(R, +, *)$ be the semiring of non-negative integers excluding 1 and under stander addition and multiplication of integers. Then $I = \{2, 3\}$ is anti-ideal of R.

Definition 3.2. Let $(R, +, *)$ be a semiring and $\emptyset \neq I \subseteq R$. If I is a left(right) anti-ideal of R and an anti-ideal of $(R, +)$. Then I is a strongly left(right) anti-ideal.

Remark 3-1. A ring X doesn't have strong anti-ideal, this is clear $as\ 0 \in X\ and\ I + X \supseteq I$.

Example 3-2. Let $(M, +, *)$ be a semiring of the even integers. Then $I = \{2\}$ is a strongly anti-ideal of M.

Example 3-3. Let $(3Z, +, *)$ be the ring of integers that are multiple of 3. Then I = {3} is an anti-ideal of $3Z$.

Proposition 3.1[14]. A ring with unity doesn't have anti-ideals.

Corollary 3.1. Every integral domain doesn't have anti-ideals.

Proof. Since every integral domain is a ring with unity, According to Proposition 3.1, there are no anti-ideals in a ring with unity. It follows that all integral domains have no anti-ideals.

Proposition 3.2. Suppose that I is a left (right) anti-ideal of A, and let $S \subseteq A$. Then $\emptyset \neq I \cap S$ is a left (right) anti ideal of A.

Proof. $\because I \neq \emptyset \Rightarrow I \cap S \neq \emptyset$.

for any *$x \in A$ and $s \in I \cap S, \Rightarrow s \in I$ since $s \in I \cap S$, and $xs \notin I$ since I is a left antiideal.*

for $s \in S \Rightarrow xs \notin S\ \forall x \in A$. Therfore, $I \cap S$ is aleft antiideal of A.

Similary we can prove that I ∩ S being a right antiideal.

Example3-4. Let $(G, +)$ is the abelian group and $(G, *)$ is the abelian semigroup. Suppose I is a left anti-ideal of $A \subseteq G$ such that $I \cup A$ is not a left anti-ideal of G.

∵ I is a left antiideal of S, for any s ∈ S and i ∈I, we have si ∉ I.

Similarly,$for\ any\ a \in A, we\ have\ sa \notin I, otherwise\ I \cup S\ would\ be\ a\ left\ antiideal.$ Take an element $x \in I \cup A$. There are two cases:

1- *If x∈ I, then for any s S, sx ∉I because I is a left antiideal.*

However, sx∈I∪A, so I∪A is not a left antiideal of S.

2- *. If x ∈ S, then for any sS, sx ∉I because I is a left antiideal.*

However, sx∈ I ∪A, so I ∪A is not a left antiideal of S.

Remark 3-2. The union of a left anti-ideal I and any subset A of $R, I \cup A$ may not be a left anti-ideal of R.

Example 3-5. Consider a semiring of non-negative even integers. Let I = {2} and J = {4} are antiideal of A. It's clear that I ∪ J is not an antiideal of A.

Rough Anti-ideals Of A Semiring

This section is crucial as it introduces the main contribution of the paper. The definitions are clear, but consider expanding on the implications of rough anti-ideals in practical applications or theoretical contexts.

Definition 4.1. Let $A \subseteq R$ and $(R, \sim)$ be a rough approximation space. If $\overline{I(A)}$ and $\underline{I(A)}$ are ideals, then $\underline{I(A)}$is called a lower, and $\overline{I(A)}$ is the upper rough ideal. $(\overline{I(A)}, \underline{I(A)})$ is called the rough ideal of R.

Definition 4.2. Let $(R, +,*), R \neq \emptyset$, be a semiring. Let N be a rough set of R. Then N is:

1- rough left anti-ideal of R if N (rx)∩N (x)= $\emptyset$.
2- rough right anti-ideal of R ifN(xr)∩N(x)=$\emptyset$.
3- rough anti-ideal of R if it is left and right anti-ideal.

Definition 4.3. Let $(R, +,*)$be a semiring and $(\emptyset \neq)$,N be a rough set of R. Then N is a strong rough antiideal if $N(a + x) \cap N(x) = \emptyset$.

Preposition 4-1. Let $(R, +,*)$be a semiring and$(N \neq \emptyset)$ is a rough set of R. Then N is a rough left anti-ideal of A if and only if $N(mx) \cap N(x) = \emptyset\ for\ all\ m \in R\ and\ x \in R.$

Proof. Let N be a rough left anti-ideal of R. Then by definition, $\forall\ m \in\ R\ and\ x \in R, \Rightarrow N\ (mx) \cap N\ (x) = \emptyset$.

On the other side, let *N (mx)∩N(x)=∅ ∀m∈R and x∈R.* Let $m \in Rand\ x \in R. Then\ N(mx) \cap N(x) = \emptyset$, this means N is a rough left anti-ideal of R.

Preposition 4-2. Let $(R, +,*)$ is a semiring and $(\emptyset \neq N)$ is a rough set of R. Then N is a strong rough anti-ideal of R if and only if *N(a+x)∩N(x)=∅ ∀a, x∈R.*

Proof. Assume that N is a strong rough anti-ideal of R. Then *N(a+x)∩N(x)=∅ , ∀a, x ∈R* (by definition)

Conversely, let *N(a+x)∩N(x)=∅ ∀a, x∈R.* Then*N (a+x)∩N (x)=∅*, this means that N is a strong rough antiideal of R.

Conclusion

This research develops the concept of rough sets in rings and makes some progress in understanding rough anti ideals in semirings, attempting to provide as much detail as possible. This will contribute to the theoretical development of rough set theory as applied to the algebraic structure of semirings. By employing identity relations and a careful analysis of the properties of right- and left-handed rough anti-ideals, we have introduced rough anti-ideals and enhanced our understanding of the associated algebraic structures. Our work seeks to advance theoretical and applied research by clarifying the properties and relationships between rough anti-ideals and semirings. Despite these advances, there are still many unexplored properties and directions. Investigating the behavior and applications of rough anti-ideals in diverse mathematical and real-world contexts offers great potential for further study. In addition, examining the connections between rough anti-ideals and other algebraic structures can lead to new discoveries and foster interdisciplinary collaboration

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