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Performance Comparison of Traditional Bootstrap and Bias-Corrected and Accelerated Methods in Constructing Confidence Intervals for Non-Normal Data: A Simulation Study

Kasem A. Farag [1 *], Mohammed A. Asselhab [2], Basmah M. Binsoud [3], Zahiya M. Abobaker [4]
[1] Department of Mathematical, Faculty of Science, University of Omar Al-Mukhtar, EL-Beyda, Libya
[2] Department of Mathematical, Faculty of Arts and sciences, Sebha University, Sebha, Libya
[3] Mathematical Department, Faculty of Arts and Sciences, Benghazi University, Qaminis, Libya
[4] Mathematical Department, faculty of Science, Derna University, Derna, Libya
[*] *Corresponding author*: *Kasem.abdinibi@omu.edu.ly*

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Abstract:
Bootstrap methods have emerged as powerful non-parametric tools for statistical inference, particularly when dealing with non-normal data distributions where traditional parametric assumptions fail. This simulation study compares the performance of traditional bootstrap and bias-corrected and accelerated (BCa) bootstrap methods in constructing confidence intervals for non-normal data. We conducted extensive Monte Carlo simulations across various non-normal distributions including exponential, chi-square, and beta distributions with different sample sizes (n = 30, 50, 100, 200). Performance metrics evaluated include coverage probability, interval width, and computational efficiency. Our results demonstrate that BCa bootstrap consistently outperforms traditional bootstrap methods, achieving coverage probabilities closer to the nominal 95% level across all tested distributions. The BCa method showed superior performance particularly for heavily skewed distributions and smaller sample sizes, with coverage probabilities ranging from 94.2% to 95.8% compared to 89.3% to 93.7% for traditional bootstrap. While BCa bootstrap requires approximately 15-20% more computational time, the improved accuracy justifies this cost. These findings provide valuable insights for practitioners dealing with non-normal data and contribute to the growing body of literature on robust statistical inference methods.

Keywords: Bootstrap methods, BCa Bootstrap, Confidence Intervals, Non-normal Data, Simulation Study, R-Programming.

مقارنة الأداء بين طريقة البوتستراب التقليدية وطريقة البوتستراب المعدّلة للتحيز والتسارع في بناء فترات الثقة للبيانات الغير الطبيعية: دراسة محاكاة

قاسم عبدالنبي فرج [1]*، محمد عبدالله السلهاب [2]، بسمة محمد بن سعود [3]، زاهية محمد ابوبكر [4]
[1] قسم الرياضيات، كلية العلوم، جامعة عمر المختار، البيضاء، ليبيا
[2] قسم الرياضيات، كلية الآداب والعلوم، جامعة سبها، سبها، ليبيا
[3] قسم الرياضيات، كلية الآداب والعلوم، جامعة بنغازي، قمينس، ليبيا
[4] قسم الرياضيات، كلية العلوم، جامعة درنة، درنة، ليبيا

الملخص
لقد برزت طرق البوتستراب كأدوات غير معلمية قوية للاستدلال الإحصائي، ولا سيما عند التعامل مع التوزيعات غير الطبيعية التي تفشل فيها الافتراضات المعلمية التقليدية. تُقارن هذه الدراسة المحاكاة بين أداء طريقتي البوتستراب التقليدية وبوتستراب التصحيح المصحح للتحيز والمسرّع (BCa) في بناء فترات الثقة للبيانات غير الطبيعية. أُجريت محاكاة مونت كارلو على نطاق واسع عبر توزيعات غير طبيعية مختلفة، تشمل التوزيع الأسي، وتوزيع كاي-تربيع، والتوزيع البيتي،

وبأحجام عينات مختلفة (n = 30 ، 50، 100، 200). وشملت مقاييس الأداء المُقيَّمة احتمال التغطية، وعرض الفترات، والكفاءة الحاسوبية. تُظهر نتائجنا أن طريقة BCa تتفوق باستمرار على طريقة البوتستراب التقليدية، حيث تحقق احتمالات تغطية أقرب إلى المستوى الاسمية البالغ 95% عبر جميع التوزيعات المختبرة. وأظهرت طريقة BCa أداءً أفضل بشكل خاص مع التوزيعات شديدة الالتواء وأحجام العينات الصغيرة، حيث تراوحت احتمالات التغطية بين 94.2% و95.8%، مقارنةً بـ 89.3% إلى 93.7% للبوتستراب التقليدي. وعلى الرغم من أن طريقة BCa تتطلب وقتًا حاسوبيًا يزيد بنسبة 15-20% تقريبًا، فإن الدقة المحسّنة تبرر هذا العبء الإضافي. تُعدّ هذه النتائج مصدرًا قيمًا من البصيرة للممارسين الذين يعملون مع بيانات غير طبيعية، وتساهم في الجسم المتنامي من الأدبيات المتعلقة بأساليب الاستدلال الإحصائي القوية.

الكلمات المفتاحية: طرق البوتستراب، البوتستراب BCa، فترات الثقة، البيانات غير الطبيعية، دراسة محاكاة، برمجة.

Introduction

Statistical inference traditionally relies on parametric methods that assume specific probability distributions, most commonly the normal distribution. However, real-world data frequently violate these assumptions, presenting challenges for accurate parameter estimation and confidence interval construction [1]. Bootstrap methods, introduced by Efron in 1979, have revolutionized statistical inference by providing non-parametric alternatives that make minimal distributional assumptions [2].

The traditional bootstrap method, also known as the percentile bootstrap, constructs confidence intervals by resampling the original data with replacement and using the empirical distribution of bootstrap statistics [3]. While this approach has proven valuable across numerous applications, it suffers from bias and skewness issues, particularly when dealing with non-normal data distributions [4]. These limitations have motivated the development of more sophisticated bootstrap variants, most notably the bias-corrected and accelerated (BCa) bootstrap method.

The BCa bootstrap, developed by Efron and subsequently refined by various researchers, addresses the shortcomings of traditional bootstrap by incorporating bias correction and acceleration parameters [5]. The bias correction adjusts for the systematic deviation between the bootstrap distribution and the true sampling distribution, while the acceleration parameter accounts for the rate of change in the standard error with respect to the parameter of interest [6].

Recent advances in computational statistics have facilitated extensive comparative studies of bootstrap methods.[7] demonstrated that BCa bootstrap provides superior coverage properties for skewed distributions, while maintaining computational feasibility for moderate sample sizes. Similarly, [8] showed that the choice of bootstrap method significantly impacts inference quality in biomedical applications where non-normal data are prevalent.

The performance of bootstrap methods is particularly crucial in modern data analysis contexts where non-normal distributions are increasingly common. Financial data often exhibit heavy tails and skewness [9], while biological and environmental data frequently follow exponential or gamma distributions [10]. In these contexts, the accuracy of confidence intervals directly impacts decision- making and scientific conclusions.

Several recent studies have investigated specific aspects of bootstrap performance. [11] focused on small sample performance, finding that BCa bootstrap maintains reasonable coverage even with samples as small as 20 observations. [12] examined computational aspects, developing efficient algorithms that reduce the computational burden of BCa bootstrap while preserving its statistical advantages.

The theoretical foundations of bootstrap methods continue to evolve. [13] provided refined asymptotic theory for BCa bootstrap, establishing conditions under which second-order accuracy is achieved. This theoretical work complements empirical studies by providing deeper understanding of when and why BCa bootstrap outperforms traditional methods.

Machine learning applications have also driven interest in robust bootstrap methods. [14] demonstrated that bootstrap confidence intervals for prediction accuracy metrics require careful consideration of the underlying data distribution, with BCa methods showing superior performance for imbalanced datasets.

Environmental statistics presents another domain where bootstrap methods prove essential.[15] applied various bootstrap methods to climate data, finding that traditional bootstrap methods often failed to capture the true uncertainty in temperature and precipitation projections, while BCa methods provided more reliable intervals.

The increasing availability of high-performance computing resources has enabled more comprehensive simulation studies. Recent work by [16] utilized parallel computing to conduct simulations with millions of replications, providing unprecedented precision in evaluating bootstrap method performance across diverse scenarios.

Quality control and industrial applications have also benefited from improved bootstrap methods. [17] showed that BCa bootstrap provides more accurate capability indices for non-normal process data, leading to better quality decisions in manufacturing environments.

Educational research has highlighted the importance of proper bootstrap method selection.[18] found that undergraduate statistics courses often oversimplify bootstrap concepts, leading to inappropriate method selection

in practice. Their work emphasizes the need for better understanding of when different bootstrap variants are appropriate.
Despite extensive research, gaps remain in our understanding of bootstrap method performance. Most comparative studies focus on specific distribution types or limited sample size ranges. Additionally, the computational trade-offs between accuracy and efficiency require further investigation, particularly as datasets continue to grow in size and complexity.
This study addresses these gaps by conducting a comprehensive simulation study comparing traditional bootstrap and BCa methods across multiple non-normal distributions and sample sizes. Our objectives are to: (1) quantify the coverage probability differences between methods across various scenarios, (2) evaluate the trade-offs between statistical accuracy and computational efficiency, (3) provide practical guidance for method selection based on data characteristics, and (4) identify scenarios where the additional complexity of BCa bootstrap is most justified.

Methodology
Our simulation study employed a comprehensive design to evaluate the performance of traditional bootstrap and BCa bootstrap methods across various non-normal data distributions. We generated data from four different distribution families: exponential (rate = 1), chi-square (df = 3), beta ($\alpha = 2$, $\beta = 5$), and log-normal ($\mu = 0$, $\sigma = 1$). Sample sizes tested included n = 30, 50, 100, and 200 to examine performance across small to moderate sample sizes.
For each combination of distribution and sample size, we conducted 10,000 Monte Carlo replications to ensure reliable estimates of coverage probabilities and other performance metrics. Bootstrap confidence intervals were constructed using B = 1,000 bootstrap samples, following recommendations from recent literature for balancing accuracy and computational efficiency.
Performance metrics included: (1) coverage probability - the proportion of confidence intervals containing the true parameter, (2) average interval width - measuring precision of estimates, (3) computational time - assessing practical feasibility, and (4) bias in interval endpoints - evaluating systematic errors.

Results
The simulation results demonstrate clear performance differences between traditional bootstrap and BCa bootstrap methods across all tested scenarios. We present comprehensive results for all four distributions (exponential, chi-square, beta, and log- normal) across four sample sizes (n = 30, 50, 100, 200).

Coverage Probability Analysis

Table 1. Coverage Probabilities (%) for 95% Confidence Intervals Across All distributions and Sample Sizes

Distribution	Sample size	Traditional Bootstrap	BCa Bootstrap	Improvement	P- value
Exponential (λ=1)	30	89.3	94.2	+4.9	0.001
	50	91.1	94.8	+3.7	0.001
	100	92.7	95.1	+2.4	0.001
	200	93.7	95.3	+1.6	0.001
Chi-square (df=3)	30	90.2	94.6	+4.4	0.001
	50	91.8	95.2	+3.4	0.001
	100	92.9	95	+2.1	0.001
	200	93.8	95.2	+1.4	0.001
Beta (α=2, β=5	30	92.1	95.0	+2.9	0.001
	50	93.2	95.4	+2.2	0.001
	100	94.1	95.6	+1.5	0.01
	200	94.5	95.7	+1.2	0.05
Log-normal (μ=0, σ=1)	30	88.9	94.7	+5.8	0.001
	50	90.6	95.1	+4.5	0.001
	100	92.3	95.4	+3.1	0.001
	200	93.5	95.8	+2.3	0.001

Table 1 demonstrates that BCa bootstrap consistently achieves coverage probabilities closer to the nominal 95% level across all distributions and sample sizes. The improvement is most pronounced for heavily skewed distributions (exponential and log- normal) and smaller sample sizes. Log-normal distribution shows the largest improvements (+5.8% at n=30), while beta distribution shows the smallest but still significant improvements. All differences are statistically significant based on two- proportion z-tests.

Interval Width Comparison

Table 2. Average Confidence Interval Widths Across All Distributions and Sample Sizes

Distribution	Sample size	Traditional Bootstrap	BCa Bootstrap	Relative difference %
Exponential (λ=1)	30	1.245	1.287	+3.4
	50	0.968	0.991	+2.4
	100	0.681	0.693	+1.8
	200	0.482	0.489	+1.5
Chi-square (df=3)	30	2.156	2.201	+2.1
	50	1.678	1.702	+1.4
	100	1.089	1.098	+0.8
	200	0.789	0.793	+0.5
Beta (α=2, β=5)	30	0.267	0.275	+3.0
	50	0.234	0.241	+3.0
	100	0.165	0.169	+2.4
	200	0.117	0.119	+1.7
Log-normal (μ=0, σ=1)	30	3.789	3.921	+3.5
	50	2.945	3.023	+2.6
	100	2.087	2.124	+1.8
	200	1.476	1.489	+0.9

Table 2 shows that BCa bootstrap intervals are consistently wider than traditional bootstrap intervals across all scenarios, with relative differences ranging from 0.5% to 3.5%. The width penalty decreases as sample size increases and is generally smaller for less skewed distributions (chi-square and beta). The modest increase in interval width represents a favorable trade-off for the substantial improvements in coverage accuracy shown in Table 1.

Computational Performance

Table 3. Computational Time Comparison (seconds per 1000 replications)

Sample size	Traditional Bootstrap	BCa Bootstrap	Time Ratio	Overhead (%)
30	2.14	2.51	1.17	17.3
50	2.98	3.48	1.17	16.8
100	4.67	5.52	1.18	18.2
200	8.23	9.81	1.19	19.2

Table 3 reveals that BCa bootstrap requires approximately 17-19% more computational time than traditional bootstrap across all sample sizes. The computational overhead remains relatively stable as sample size increases, ranging from 16.8% to 19.2%. This consistent overhead makes BCa bootstrap computationally feasible even for larger datasets, with the performance benefits clearly justifying the additional computational cost.

Distribution-Specific Performance Summary

Table 4. Performance Summary by Distribution Type (Averaged Across All Sample Sizes)

Distribution	Skewness	Traditional Bootstrap	BCa Bootstrap	Average Improvement	Width Penalty (%)
Log-normal	6.18	91.3	95.3	+4.0	2.2
Exponential	2.00	91.7	94.9	+3.2	2.3
Chi-square	1.63	92.2	95.0	+2.8	1.2
Beta	0.57	93.5	95.4	+1.9	2.5

Table 4 demonstrates a clear relationship between distribution skewness and BCa bootstrap advantage. The most heavily skewed distribution (log-normal) shows the largest coverage improvement (+4.0%), while the least skewed distribution (beta) shows the smallest improvement (+1.9%). However, even for the beta distribution, BCa bootstrap provides statistically significant and practically meaningful improvements. The width penalty remains modest across all distributions, ranging from 1.2% to 2.5%.

Discussion

Our simulation results provide compelling evidence for the superior performance of BCa bootstrap methods when constructing confidence intervals for non-normal data. These findings align with and extend previous research in several important ways.

The coverage probability results strongly support the theoretical advantages of BCa bootstrap documented by [1]. Our finding that BCa bootstrap achieves coverage probabilities consistently above 94% across all tested scenarios confirms the method's second-order accuracy properties. This is particularly notable for heavily skewed distributions like exponential and log-normal, where traditional bootstrap methods showed substantial under coverage, achieving only 88-92% coverage.

The performance differences we observed are consistent with the theoretical work of [4], who predicted that bias correction would be most beneficial for skewed distributions. Our results extend their findings by quantifying these benefits across multiple distribution types and sample sizes, providing practical guidance for method selection.

Compared to the study by [7], our results show similar patterns but with slightly better BCa performance, possibly due to our use of 1,000 bootstrap samples rather than their 500. This suggests that the number of bootstrap replications significantly impacts BCa performance, particularly for smaller sample sizes.

The computational overhead we documented (17-19% increase) is lower than reported by [12], who found 25-30% increases. This difference may reflect improvements in implementation efficiency or differences in computing environments. Importantly, our results demonstrate that this computational cost remains manageable and relatively constant across sample sizes.

Our interval width analysis reveals an important trade-off: BCa intervals are 0.8-3.4% wider than traditional bootstrap intervals, but this small precision cost yields substantial improvements in coverage accuracy. This finding supports the conclusions of [11] that the accuracy gains justify the modest reduction in precision.

The practical implications of our findings are significant for applied statisticians. In scenarios involving non-normal data, particularly with small to moderate sample sizes, BCa bootstrap should be preferred despite its computational overhead. The coverage improvements we documented translate directly to more reliable statistical inference and better-calibrated uncertainty quantification.

Conclusion

This comprehensive simulation study provides strong empirical evidence supporting the superiority of BCa bootstrap methods over traditional bootstrap approaches for constructing confidence intervals with non-normal data. Our results demonstrate that BCa bootstrap consistently achieves coverage probabilities closer to nominal levels across all tested distributions and sample sizes, with particularly pronounced advantages for heavily skewed distributions.

The practical implications of our findings are clear: despite the 17-19% computational overhead, BCa bootstrap should be the preferred method when dealing with non-normal data, especially when accurate uncertainty quantification is critical. The modest increase in interval width (0.8-3.4%) represents a favorable trade-off for substantially improved coverage accuracy.

Future research directions include extending these comparisons to other bootstrap variants such as studentized bootstrap and examining performance with extremely small samples ($n < 30$). Additionally, investigating the performance of these methods with multivariate non-normal data and complex survey designs would provide valuable insights for applied researchers.

Our implementation code in both R provides practitioners with readily applicable tools for implementing these methods in their own analyses, contributing to the broader adoption of robust statistical inference techniques in data analysis workflows.

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